Universidad Simón Bolívar
Departamento de Electrónica y Circuitos
EC3514
Prof. Cecilia Murrugarra
Formulario: Robótica

1. Identidades Trigonométricas

$cos^2(\theta) + sen^2(\theta) = 1$
$cos^2(\theta) - sen^2(\theta) = cos(2\theta)$
$cos^2(\theta) = \frac{1}{2}[1 + cos(2\theta)]$
$sen^2(\theta) = \frac{1}{2}[1 - cos(2\theta)]$
$sen(\alpha \pm \beta) = sen(\alpha)cos(\beta) \pm cos(\alpha)sen(\beta)$
$cos(\alpha \pm \beta) = cos(\alpha)cos(\beta) \mp sen(\alpha)sen(\beta)$
$c^2 = a_1^2 + a_2^2 - 2a_1a_2cos(\alpha)$

2. Cinemática

$$Rot_0^1 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}$$

$$Rot_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha \\ 0 & S_\alpha & C_\alpha \end{bmatrix}$$

$$Rot_{y,\alpha} = \begin{bmatrix} C_\alpha & 0 & S_\alpha \\ 0 & 1 & 0 \\ -S_\alpha & 0 & C_\alpha \end{bmatrix}$$

$$Rot_{z,\alpha} = \begin{bmatrix} C_\alpha & -S_\alpha & 0 \\ S_\alpha & C_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.1. Parámetros Denavit-Hartenberg

$\theta_i = \widehat{X_{i-1}, X_i} / Z_{i-1}$
$d_i =$ distancia $(O_{i-1} \to X_i \cap Z_{i-1})/Z_{i-1}$
$\alpha_i = \widehat{Z_{i-1}, Z_i} / X_i$
$a_i =$ distancia $(X_i \cap Z_{i-1} \to O_i)/X_i$

$$A_{D-H} = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i}C_{\alpha_i} & S_{\theta_i}S_{\alpha_i} & a_iC_{\theta_i} \\ S_{\theta_i} & C_{\theta_i}C_{\alpha_i} & -C_{\theta_i}S_{\alpha_i} & a_iS_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.2. Velocidad

$$\dot{X} = J(q)_{6xn}\dot{q}_{6xn} = \begin{bmatrix} \overrightarrow{V}_0^n \\ \overrightarrow{\omega}_0^n \end{bmatrix} = \begin{bmatrix} J_L(q) \\ J_\omega(q) \end{bmatrix}_{6xn} * \overrightarrow{\dot{q}}_{nx1}$$

2.2.1. Método Analítico

$\dot{X}_T = \frac{d}{dt}\Big(d_0^n(q_1, \ldots, q_n)\Big);$
$\dot{R}_0^n R_0^{nT} = S(\omega)$

$$S(\omega) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

2.2.2. Método Cinemático

$$Art.Prismatica = J_i = \begin{bmatrix} [Z_{i-1}]_{3x1} \\ [0]_{3x1} \end{bmatrix}$$

$$Art.Rotacional = J_i = \begin{bmatrix} Z_{i-1} \times (0_n - 0_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

3. Dinámica

$Q_i = \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i}$

$L = \sum_{i=1}^n K_i - \sum_{i=1}^n U_i$

$K = \frac{1}{2}m \mid \vec{V} \mid^2 + \frac{1}{2}\vec{\omega}^T I \vec{\omega}$

$U = mgh$

$\vec{V} = \dot{\vec{r}}_{cm} + \vec{\omega} \times \vec{r}_{cm}$